FIRST YEAR HIGHER SECONDARY EXAMINATION SAY/IMP SEPTEMBER 2016

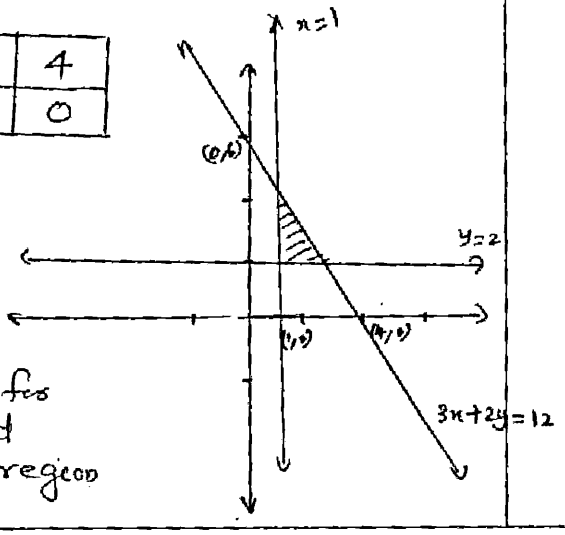
(Scheme of Valuation)

Subject : Mathematics (Science) **Code No. 418**

Qn. No		Scoring Indicators	Split Score	Total Score
1.	a)	$A' = \{1, 3, 5, 7\}$ $B' = \{1, 3, 5, 6, 7\}$	½ ½ } 1	1
	b)	$A \cup B = \{2, 4, 6, 8\}$ $(A \cup B)' = \{1, 3, 5, 7\}$ $A' \cap B' = \{1, 3, 5, 7\}$ $(A \cup B)' = A' \cap B'$	½ ½ 1 } 2	2
	c)	$A \cap B = \{2, 4, 8\}$ $(A \cap B)' = \{1, 3, 5, 6, 7\}$ $A' \cup B' = \{1, 3, 5, 6, 7\}$ $\therefore (A \cap B)' = A' \cup B'$	½ ½ 1 } 2	2
2.	a)	False	1	1
	b)	$R = \{(2, 2^3)(3, 3^3)(5, 5^3)(7, 7^3)\}$ $= \{(2, 8)(3, 27)(5, 125)(7, 343)\}$ Remark: Give ½ score for each pair	2	2
	c)	f is a relation Because f is a subset of $A \times B$ OR f is not a function Because the element '2' has two images	1½ 1½ 1½ 1½	3
3.	a)	(iv) or $\frac{121\pi}{540}$	1	1
	b)	$LHS = 3 \times \frac{1}{2} \times 2 - 4 \sin(\pi - \pi/6) \times \frac{1}{2}$ $= 3 - 4 \sin \pi/6$ $= 3 - 4 \times \frac{1}{2}$ $= 3 - 2 = 1 = RHS$	1 ½ ½ } 2	2

Remark. For two correct values give ½ score.

Qn.No		Scoring Indicators	Split Score	Total Score
	(c)	Given $\sin 6x + \sin 2x - \sin 4x = 0$	½	
		$\Rightarrow 2\sin 4x \cos 2x - \sin 4x = 0$	½	
		$\Rightarrow \sin 4x (2\cos 2x - 1) = 0$	½	
		$\sin 4x = 0$ or $2\cos 2x - 1 = 0$	½	4
		$4x = n\pi$ or $2\cos 2x = 1$	½ + ½	
		$\cos 2x = \frac{1}{2} = \cos \pi/3$		
		$x = \frac{n\pi}{4}$ or $2x = 2n\pi \pm \pi/3$ $n \in Z$	½ + ½	
		$x = n\pi \pm \pi/6$		
		Remark: Only for.		
		$\sin x = 0$ then $x = n\pi$ (½)		
		$\cos x = \cos y$ then $x = 2n\pi \pm y$ (½)		
		$\sin x + \sin y = 2\sin \frac{x+y}{2} \cos \frac{x-y}{2}$ (½)		
4	(a)	$P(1) = x^1 - y^1$ is divisible by $(x-y)$, which is true	1	
	b)	Suppose $P(k)$ is true,		
		$P(k)$: $x^k - y^k$ is divisible by $(x-y)$	1	
		Now consider		
		$P(k+1)$: $x^{k+1} - y^{k+1}$		
		$= x^k \cdot x - y^k \cdot y$	½	
		$= x^k \cdot x - x^k \cdot y + x^k \cdot y - y^k \cdot y$	½	2
		$= x^k(x-y) + y(x^k - y^k)$	½	
		which is divisible by $(x-y)$	½	4
		$\therefore$ By PMI it is true for all Natural number		

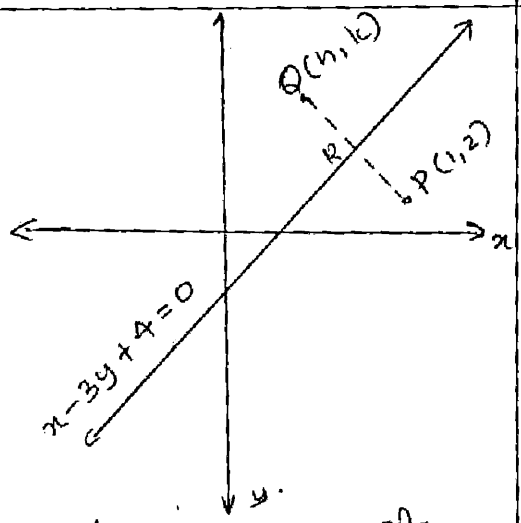
Qn.No		Scoring Indicators	Split Score	Total Score
5.	(a)	Real Part $= -3$ Imaginary $= \sqrt{7}$	½ + ½	1
	b)	$a + ib = 1 + i\sqrt{3} \Rightarrow a = 1, b = \sqrt{3}$		
		$r = \sqrt{a^2+b^2} = 2$	½ + ½	
		$r\cos\theta = 1 \Rightarrow \cos\theta = 1/2$ $r\sin\theta = \sqrt{3} \Rightarrow \sin\theta = \sqrt{3}/2$	½	2.
		or $\tan\theta = \sqrt{3}$ $\therefore \theta = \pi/3$	½	
	c)	$x^2 - 2x + 3 = 0$ $a = 1 \quad b = -2 \quad c = 3$ $\therefore x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{2 \pm \sqrt{4-12}}{2}$	1	
		$= \frac{2 \pm \sqrt{-8}}{2}$	½	2.
		$= \frac{2 \pm i\sqrt{8}}{2}$ or $\frac{2 \pm i2\sqrt{2}}{2}$ $= 1 \pm i\sqrt{2}$	½	
6.	(a)	(iv) or $3x - 1 \geq 5 \quad x \in R$	1	1
	(b)	<table><tr><td>x</td><td>0</td><td>4</td></tr><tr><td>y</td><td>6</td><td>0</td></tr></table>  Give (1) Score for each line and (1) for feasible region		4

Qn.No		Scoring Indicators	Split Score	Total Score
7.	a)	(iv) or 360	1	1
	b)	No. of ways $= {}^{n}C_{r} = {}^{5}C_{3}$	1½	2
		$= 10$	½	
	c)	${}^{2n}C_{3} = 11 \cdot {}^{n}C_{3}$		
		$\frac{2n!}{(2n-3)!\,3!} = 11 \cdot \frac{n!}{(n-3)!\,3!}$	1	3
		$\Rightarrow 2n(2n-1)(2n-2) = 11 \cdot n(n-1)(n-2)$	1	
		$\Rightarrow 8n - 4 = 11n - 22$	1	
		$\Rightarrow 3n = 18 \Rightarrow n = 6$		
		Remark: Only for (b) or (c) ${}^{n}C_{r} = \frac{n!}{(n-r)!\,r!}$ give (½) Score		
		Remark: 7(b) using ${}^{2}C_{1}\,{}^{3}C_{2} + {}^{2}C_{2}\,{}^{3}C_{1} + {}^{2}C_{0}\,{}^{3}C_{3} = 10$ give (2) score.		
OR				
7	(a)	1	1	1
	(b)	${}^{2n}C_{3} : {}^{n}C_{3} = 12 : 1$		
		$\Rightarrow \frac{{}^{2n}C_{3}}{{}^{n}C_{3}} = \frac{12}{1}$	½	
		$\frac{\frac{2n!}{(2n-3)!\,3!}}{\frac{n!}{(n-3)!\,3!}} = \frac{12}{1}$	½	2
		$\frac{2n(2n-1)(2n-2)}{n(n-1)(n-2)} = \frac{12}{1}$	½	
		$4n = 20 \Rightarrow n = 5$	½	

Qn.No		Scoring Indicators	Split Score	Total Score
	c.	Arrangement of letters is of the form $\_E\_U\_A\_I\_O\_$ There are five vowels and three consonents. No of arrangements of Vowels $= 5P_5 = 5! = 120$	1	
		No of arrangements of three consonents in 6 places $= 6P_3 = 120$	1	3
		$\therefore$ Required number of arrangements $= 5P_5 \times 6P_3 = 120 \times 120 = 14400$	1	
		Remark: Only for ${}^nP_r = \frac{n!}{(n-r)!}$ ($\frac{1}{2}$) score.		
8	a)	$(a+b)^4 = a^4 + 4C_1 a^3 b + 4C_2 a^2 b^2 + 4C_3 a b^3 + b^4$ OR $\Rightarrow a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$	1	1
	b)	$(\sqrt{5}+\sqrt{6})^4 = (\sqrt{5})^4 + 4(\sqrt{5})^3\sqrt{6} + 6(\sqrt{5})^2(\sqrt{6})^2 + 4\sqrt{5}(\sqrt{6})^3 + (\sqrt{6})^4$	1	
		$(\sqrt{5}-\sqrt{6})^4 = (\sqrt{5})^4 - 4(\sqrt{5})^3\sqrt{6} + 6(\sqrt{5})^2(\sqrt{6})^2 - 4\sqrt{5}(\sqrt{6})^3 + (\sqrt{6})^4$	1	3
		$(\sqrt{5}+\sqrt{6})^4 + (\sqrt{5}-\sqrt{6})^4 = 2(\sqrt{5})^4 + 12(\sqrt{5})^2(\sqrt{6})^2 + 2(\sqrt{6})^4$	$\frac{1}{2}$	
		$= 2\times25 + 12\times5\times6 + 2\times36$ $= \underline{482}$.	$\frac{1}{2}$.	

Remark: Correct answer using the formula $(a+b)^4 + (a-b)^4$ give 3 score

Qn.No		Scoring Indicators	Split Score	Total Score
9	a)	(iv) or $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$	1	1
	b)	$a = 1. \quad r = \frac{2}{3} \quad n = 5$	½	
		$S_n = \frac{a(1-r^n)}{(1-r)}$	½	
		$= \frac{1\left[1-\left(\frac{2}{3}\right)^5\right]}{1-\frac{2}{3}}$	½	2
		$= \left(\frac{243-32}{243}\right) \times 3 = \frac{211}{81}$	½	
	c)	$S_n = 0.6 + 0.66 + 0.666 + \cdots$ n terms		
		$= 6(0.1 + .11 + .111 + \cdots$ n terms$)$	½	
		$= \frac{6}{9}\left[.9 + .99 + .999 + \cdots\right.$ n terms$)$	½	
		$= \frac{6}{9}\left[\left(1-\frac{1}{10}\right) + \left(1-\frac{1}{10^2}\right) + \cdots\right.$ n terms$]$	½	
		$= \frac{6}{9}\left[n - \frac{1}{10}\left(1 + \frac{1}{10} + \frac{1}{100} + \cdots\right.\right.$ n terms$)$		2.
		$= \frac{6}{9}\left[n - \frac{1}{10} \times \frac{10}{9}\left(1 - \frac{1}{10^n}\right)\right]$	½.	
		$= \frac{6}{81}\left[9n - \left[1 - \frac{1}{10^n}\right)\right]$		
10.	a)	(iv) or 2/3	1	1
	b)	$y - y_1 = m(x - x_1)$	½	
		$m = -2$; $(x_1, y_1) = (-3, 0)$	½	2
		eqn is $y - 0 = -2(x - -3)$	½	
		$y = -2x - 6$	½	
		Remark: Using intercept form $y = m(x - d)$ $m = -2$, $d = -3$		
		give (2) score.		

Qn.No		Scoring Indicators	Split Score	Total Score
	(c)			
		Let Q(h,k) be the image of P(1,2) $x-3y+4=0$ —(1)	1/2	
		bisect PQ at R		
		$R\left(\frac{h+1}{2}, \frac{k+2}{2}\right)$	1/2	3
		R' lies on (1)		
		$\frac{h+1}{2} - 3\left(\frac{k+2}{2}\right)+4=0$	1/2	
		$\Rightarrow h-3k+3=0$ —(2)	1/2	
		Slope of PQ = $\frac{-1}{\text{Slope of (1)}}$		
		$\frac{k-2}{h-1} = \frac{-1}{\frac{-1}{-3}} \Rightarrow 3h+k=5$ (3)	1/2	
		$\therefore h=\frac{6}{5}$ and $k=\frac{7}{5}$	1/2	
		Image $\left(\frac{6}{5}, \frac{7}{5}\right)$		
		Remark: For correct answer using any alternative method give (3) score.		

Qn.No		Scoring Indicators	Split Score	Total Score
11	a)	$\frac{x^2}{9} - \frac{y^2}{16} = 1 \quad a=3 \quad b=4$		
		Foci are $(\pm ae, 0)$ or $(\pm c, 0)$	½	
		$= (\pm 5, 0)$	½	
		Vertices are $(\pm a, 0) = (\pm 3, 0)$	½ + ½	
		eccentricity, $e = \frac{\sqrt{a^2+b^2}}{a}$ or $\frac{c}{a}$	½	4
		$= \frac{5}{3}$	½	
		length of latus rectum $= \frac{2b^2}{a}$	½	
		$= \frac{32}{3}$	½	
		Remark: Only for $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ or $\frac{x^2}{9} - \frac{y^2}{16} = 1$ (½) score		
12	(a)	True	1	1
	(b)	let $P(x, y, z)$ be the point such that $PA = PB$	½	
		$\sqrt{(x-3)^2 + (y-4)^2 + (z+5)^2} =$	½	
		$\sqrt{(x+2)^2 + (y-1)^2 + (z-4)^2}$	½	2
		$\Rightarrow 10x + 6y - 18z - 29 = 0$	½	
		Remark: For distance formula give (½) score		

Qn.No		Scoring Indicators	Split Score	Total Score
13.	a	$\sec^2 x$	1	1
	b)	$\lim_{x\to 0} \frac{e^{3x}-1}{x} = 3 \lim_{x\to 0} \frac{e^{3x}-1}{3x}$	1	2
		$= 3 \times 1 = 3$	1	
		Remark : Only $\lim_{x\to 0} \frac{e^x-1}{x} = 1$ give (½) score		
	c)	$\frac{d}{dx} f(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$	½	
		$\frac{d}{dx} \cos x = \lim_{h\to 0} \frac{\cos(x+h) - \cos x}{h}$	½	
		$= \lim_{h\to 0} \frac{-2 \sin\left(\frac{2x+h}{2}\right) \sin\frac{h}{2}}{h}$	½	3
		$= \lim_{h\to 0} \frac{-2 \sin\left(x+\frac{h}{2}\right) \sin(h/2)}{\left(\frac{h}{2}\right) \times 2}$	½	
		$= -\sin x$	1	
		Remark (i) $\frac{d}{dx}\cos x = -\sin x$ (1) Score		
		OR		
13	(a)	$\cos x$	1	1
	(b)	$\lim_{x\to 0} \frac{\sin ax}{\sin bx} = \lim_{x\to 0} \frac{\left(\frac{\sin ax}{ax}\right) ax}{\left(\frac{\sin bx}{bx}\right) bx}$	1	2
		$= \frac{a}{b} \lim_{x\to 0} \frac{\frac{\sin ax}{ax}}{\frac{\sin bx}{bx}} = \frac{a}{b}$	½+½	

Remark: Only for $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$ (½) score.

Qn.No		Scoring Indicators	Split Score	Total Score
	c)	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$	½	
		$\frac{dy}{dx} = \frac{(3+7\cos x)(5\cos x) - (4+5\sin x)(-7\sin x)}{(3+7\cos x)^2}$	1½	
		$= \frac{15\cos x + 35\cos^2 x + 28\sin x + 35\sin^2 x}{(3+7\cos x)^2}$	½	3
		$= \frac{15\cos x + 28\sin x + 35}{(3+7\cos x)^2}$	½	
14	a)	$\sqrt{2}$ is a complex no OR It is false that $\sqrt{2}$ is not a complex number.	1	1
	b)	Assume that $\sqrt{11}$ is rational	½	
		$\sqrt{11} = \frac{a}{b} \Rightarrow 11 = \frac{a^2}{b^2}$ $\Rightarrow a^2 = 11b^2$ $\Rightarrow$ 11 divides a	½	
		let $a = 11c$ $a^2 = 121c^2$ $11b^2 = 121c^2$ $b^2 = 11c^2$. $\therefore$ 11 divides b 11 divides both a and b. $\therefore$	½ ½. ½	3
		which is a contradiction $\therefore \sqrt{11}$ is irrational	½	

Qn.No		Scoring Indicators	Split Score	Total Score
15.	a)	$\sqrt{8}$ or $2\sqrt{2}$	1	1
	b)			

Marks	x_i	f_i	$f_i x_i$	x_i^2	$f_i x_i^2$
0-10	5	5	25	25	125
10-20	15	8	120	225	1800
20-30	25	15	375	625	9375
30-40	35	16	560	1225	19600
40-50	45	6	270	2025	12150
		50	1350		43050

1

$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{1350}{50} = 27$ — $\frac{1}{2}+\frac{1}{2}$

$\sigma = \sqrt{\frac{\sum x_i^2 f_i}{N} - (\bar{x})^2}$ — $\frac{1}{2}$

$= \sqrt{\frac{43050}{50} - 27^2}$

$= 11.49$ — $\frac{1}{2}$ — 4

$C.V = \frac{\sigma}{\bar{x}} \times 100$ — $\frac{1}{2}$

$= \frac{11.49 \times 100}{27} = 42.56$ — $\frac{1}{2}$.

Remarks

For correct answer using any other alternative method give (4) score.

Qn.No		Scoring Indicators	Split Score	Total Score
16.	a)	0.2	1	1
	b)	$P(NSS) = \frac{32}{60}$ $P(NCC) = \frac{30}{60}$ $P(NSS \cap NCC) = \frac{24}{60}$	½	
	(i)	$P(NSS \cup NCC) =$ $= P(NSS) + P(NCC) - P(NSS \cap NCC)$ $= \frac{32}{60} + \frac{30}{60} - \frac{24}{60}$ $= \frac{38}{60}$	½ ½ 1½	2
	(ii)	$P(NSS' \cap NCC')$ $= P(NSS \cup NCC)'$ $= 1 - P(NSS \cup NCC)$ $= 1 - \frac{38}{60}$ $= \frac{22}{60}$	½ ½ ½ ½	2
		Remark: Only for $n(NSS) = 32$ $n(NCC) = 30$ $n(NSS \cap NCC) = 24$ } (½) score.		